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ANTICIPATED ORBITAL PERTURBATIONS OF SATELLITE 1959 DELTA TWO

by

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1. Introduction

As announced by the Smithsonian Astrophysical Observatory in a press release dated August 21, 1959 (see Appendix), the orbit of Satellite 1959 Delta Two, the "Paddle-Wheel," is significantly affected by lunar and solar perturbations. Thus, Satellite 1959 Delta Two is unique among satellites launched to date and we believe it worthwhile to publish the present analysis to provide a basis for anticipating its orbital behavior.

Starting from the orbital elements provided by the National Aeronautics and Space Administration for September 3, 1959, we have carried out numerical integration to predict the future behavior of this satellite. These integrations, based on variation of parameters, are preliminary in nature. Techniques for a more precise analysis of the orbit are in preparation.

The equations employed are outlined in Section 2. The technique of integration was a simple one taking the semi-major axis as independent variable.

2. The Perturbation Equations

a) Solar and Lunar Perturbations. -- Variations of the orbital eccentricity due to the moon and the sun are expressed by the equation (Kozai, 1959),

$$\frac{de}{dt} = -\frac{\sqrt{1-e^2}}{na^2 e}\frac{\partial R}{\partial \omega}, \tag{1}$$

where the principal terms of R are:

$$\begin{aligned} R = \frac{15}{8} e^2 a^2 \Big[& n_\odot^2 \cos^4\frac{i}{2}\cos^4\frac{\varepsilon}{2} \cos 2(\lambda_\odot - \omega) \\ & + \frac{1}{2}(n_☾^2 m_☾ + n_\odot^2)\sin^2 i\,(1 - \frac{3}{2}\sin^2\varepsilon)\cos 2\omega \\ & + \frac{1}{2} n_☾^2 m_☾ \Big\{ \sin^2\varepsilon \cos^4\frac{i}{2}\cos 2(\omega + \Omega - \Omega_☾) \\ & \qquad - \sin i \cos^2\frac{i}{2}\sin 2\varepsilon \cos(2\omega + \Omega - \Omega_☾)\Big\} \\ & + \frac{1}{2} n_\odot^2 \Big\{ \sin^2\varepsilon\cos^4\frac{i}{2}\cos 2(\omega + \Omega) \\ & \qquad - \sin i \cos^2\frac{i}{2}\sin 2\varepsilon\cos(2\omega + \Omega)\Big\} \\ & + n_\odot^2 \sin i \cos^2\frac{i}{2}\sin\varepsilon\cos^2\frac{\varepsilon}{2}\cos(2\lambda_\odot - 2\omega - \Omega)\Big]. \end{aligned} \tag{2}$$

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Here, $n_{\odot}$ is the mean motion of the sun; $n_{☾}$, that of the moon; ε, the obliquity; $\lambda_{\odot}$, the mean longitude of the sun; $m_{\odot}$, the mass of the moon (the unit is the mass of the earth); and $\Omega_{☾}$, the longitude of the ascending node of the moon with respect to the earth's equator. On the right side of equation (2), i, Ω and ω may be considered not to be affected by the moon and the sun.

As the semi-major axis does not change rapidly due to the moon and the sun, the variation of the perigee distance, q, is

$$\frac{dq}{dt} = - a \frac{de}{dt} \quad . \tag{3}$$

b) Oblateness Perturbations. -- The rates of change of the argument of perigee and the right ascension of the ascending node were computed from the usual first-order equations,

$$\frac{d\omega}{dN} = \frac{J}{p^2} (2-2.5 \sin^2 i), \tag{4}$$

$$\frac{d\Omega}{dN} = - \frac{J}{p^2} \cos i,$$

where N is the revolution number and $p = a(1-e^2)$. The value $J = .0016230$ was employed as well as the equation,

$$P = .0586745\ a^{3/2} \tag{5}$$

relating the orbital period in days to the semi-major axis in units of 6378.388 km.

c) Atmospheric-drag Perturbations. -- The effects of atmospheric drag on the orbital energy and the perigee height were introduced in the following manner.

The effect on perigee height was computed from the approximation,

$$\frac{dq}{dQ} = \frac{H}{ae} \quad , \tag{6}$$

where H is the atmospheric scale height. This relation can be transformed to the equation,

$$\frac{dq}{da} = \frac{2}{Kae+1} \tag{7}$$

where $K = H^{-1}$.

This approximation is valid for orbits of high eccentricity, but clearly is very poor for low eccentricities. We do not feel that the present calculations would be significantly affected by a more precise computation of dq/da.

The drag effect on orbital period was evaluated with a formula derived to be valid for a wide range of eccentricities.

The loss of energy due to drag may be written as

$$du = \frac{C_D}{2} A \rho w^2 ds \quad , \tag{8}$$

where C_D is the drag coefficient, A is the satellite's effective cross-sectional area, ρ is the atmospheric density, w is the orbital velocity, and ds is the differential of distance along the orbit.

For the orbital velocity we substituted the value at perigee,

$$w^2 = k^2 \frac{1+e}{a(1-e)} \quad .$$

For ds we used the equations,

$$\frac{ds}{dv} = r\left(1 + \left(\frac{1}{r}\frac{dr}{dv}\right)^2\right)^{1/2} \quad , \tag{9}$$

$$\frac{dr}{dv} = r \frac{\sin v}{1+e \cos v}$$

On the assumption that v, the true anomaly, is much less than one radian in the region of significant drag, we derived the equation,

$$ds = a(1-e)\left(1 + \frac{e}{1+e} v^2\right) dv \ . \tag{10}$$

We further assumed that the atmospheric density above perigee can be represented by an exponential function of height, writing $\rho(v) = \rho(q) \exp\left(-K(q)\left(r(v) - q\right)\right)$.

We then found that

$$du = \frac{C_D}{2} A \rho(q) k^2 (1+e)\left(1 + \frac{e}{1+e} v^2\right) \exp\left(\frac{-Kqe}{2(1+e)} v^2\right) dv \ . \tag{11}$$

Integrating over true anomaly and employing the relation

$$da = \frac{a^2}{mk^2} du \quad ,$$

where m is the satellite mass, leads to the following expression for Δa, the change of semi-major axis per revolution,

$$\Delta a = \sqrt{\frac{\pi}{2}}\, C_D \frac{A}{m} \rho(q) \frac{(1+e)^{3/2}}{\sqrt{Kqe}} a^2 \left(1 + \frac{1}{Kq}\right) \ . \tag{12}$$

We note that the integration around an orbit is performed with an exponential atmosphere. However, because of the wide range of perigee height produced by the lunar and solar perturbations it was not sufficiently accurate to assume that the entire atmosphere was isothermal.

We employed the following formulae for ρ (q) and K(q):

$$\rho(q) = 7.94 \times 10^{-11} \exp(-161.55(q-1) + 2.3029 \exp(-138.46(q-1) + 2.8)) ,$$

$$K(q) = 161.55 + 318.86 \exp(-138.46(q-1) + 2.8) , \tag{13}$$

where ρ is in gm/cm^3, q is in units of 6378.388 km, and K is in km^{-1}.

Table 1 lists the values of $\log_{10}\rho$ derived from this model and from the Smithsonian Astrophysical Observatory Interim Model Atmosphere No. 4.

Table 1

ATMOSPHERIC DENSITIES

Height	$\log_{10}\rho$ (gm/cm^3)	
(Km)	Approximation	Model 4
150	-11.12	-11.08
180	-11.75	-11.77
210	-12.24	-12.29
240	-12.65	-12.73
270	-13.02	-13.09

3. Numerical Results

The perturbed orbit has been calculated from the following initial orbit:

Epoch 1959 September 3.150 GMT

$a = 4.3446$
$e = .7604$
$i = 47.10$
$\omega = 41.66$
$\Omega = 55.62$

The results are summarized in Figures 1, 2, and 3. The arrows indicate the date of launching. In these figures the integrations for two values of A/m are shown, in order to indicate the sensitivity of the orbit to variations of atmospheric density and effective area of the satellite.

The semi-annual variation of perigee distance is produced by the solar perturbation and does not drastically affect the satellite lifetime. The lunar perturbation produces the rapid drop of perigee around 600 days after September 3, 1959 and ends the life of the satellite.

The two "acceleration" curves of da/dN, plotted logarithmetically, are essentially mirror images of the perigee height.

Dr. Don Lautman has derived an acceleration of

$$\frac{da}{dN} = -1.5 \times 10^{-4} \text{ earth radii}$$

from observations of this satellite during the first month of flight, and this value falls between the plotted curves.

References

KOZAI, Y.
1959. On the effects of the sun and the moon upon the motion of a close earth satellite. Special Rep. No. 22, Smithsonian Astrophys. Obs., p. 7.

APPENDIX

Reduction of "Paddle-Wheel" Satellite's Lifetime

Dr. Yoshihide Kozai, Astronomer of the Smithsonian Astrophysical Observatory, has predicted that the lifetime of the recently launched "paddle-wheel" satellite, Explorer VI, will be greatly reduced by perturbations of the moon's orbit. Perturbations are irregularities in the motion or orbit of a heavenly body caused by some force other than that which determines its usual path.

The "paddle-wheel" satellite's apogee of 25,000 miles is the greatest distance from the earth ever reached by any artificial satellite. Based on his prior investigation of the relation between satellite orbital irregularities and lunar perturbations, Dr. Kozai has determined that Explorer VI will be affected by the moon's gravitational field. This, in turn, will affect the perigee, drawing it down into the earth's atmosphere, leading to a significant increase in the air-drag effect.

Because of the severity of the moon's effect, the apogee of Explorer VI will be little affected by the atmospheric density at high altitudes responsible for bringing down other artificial satellites. Hence, the "paddle-wheel" will be the first satellite to have its lifetime appreciably disturbed by the moon.

The sun's attraction adds a small contribution to the shortening of this object's lifetime.

Dr. Charles A. Whitney, Physicist in charge, Research and Analysis, Smithsonian Astrophysical Observatory, has performed a detailed numerical integration of Dr. Kozai's perturbation equations in combination with equations describing the effects of air-drag.

These calculations, performed with a high-speed electronic computer, show that although the lifetime of the satellite would be more than two decades in the absence of the moon, the lunar perturbations reduce the expected lifetime to about two years.

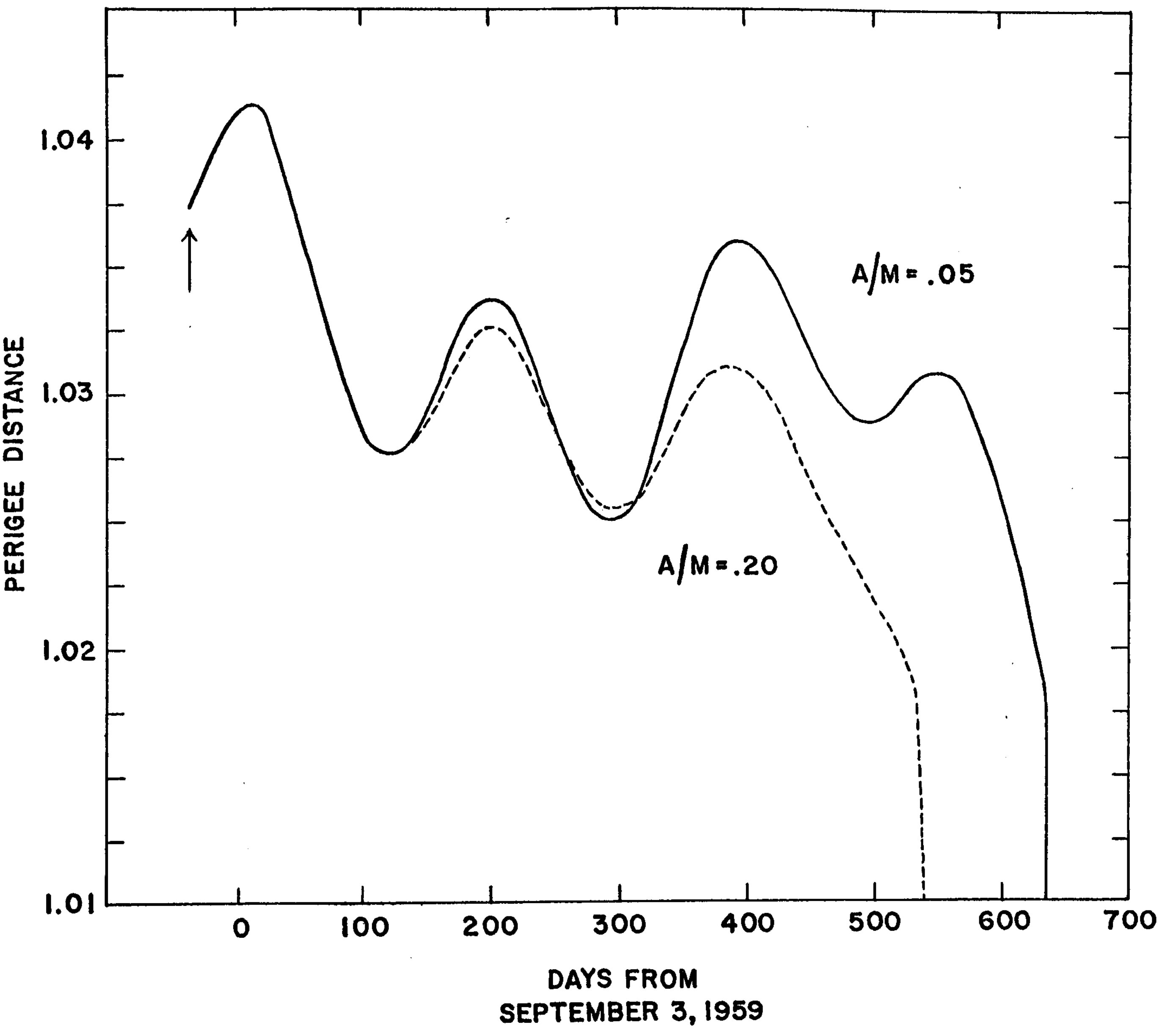

FIGURE 1. - EXPECTED PERIGEE DISTANCE OF 1959 DELTA TWO MEASURED IN UNITS OF EARTH RADII.

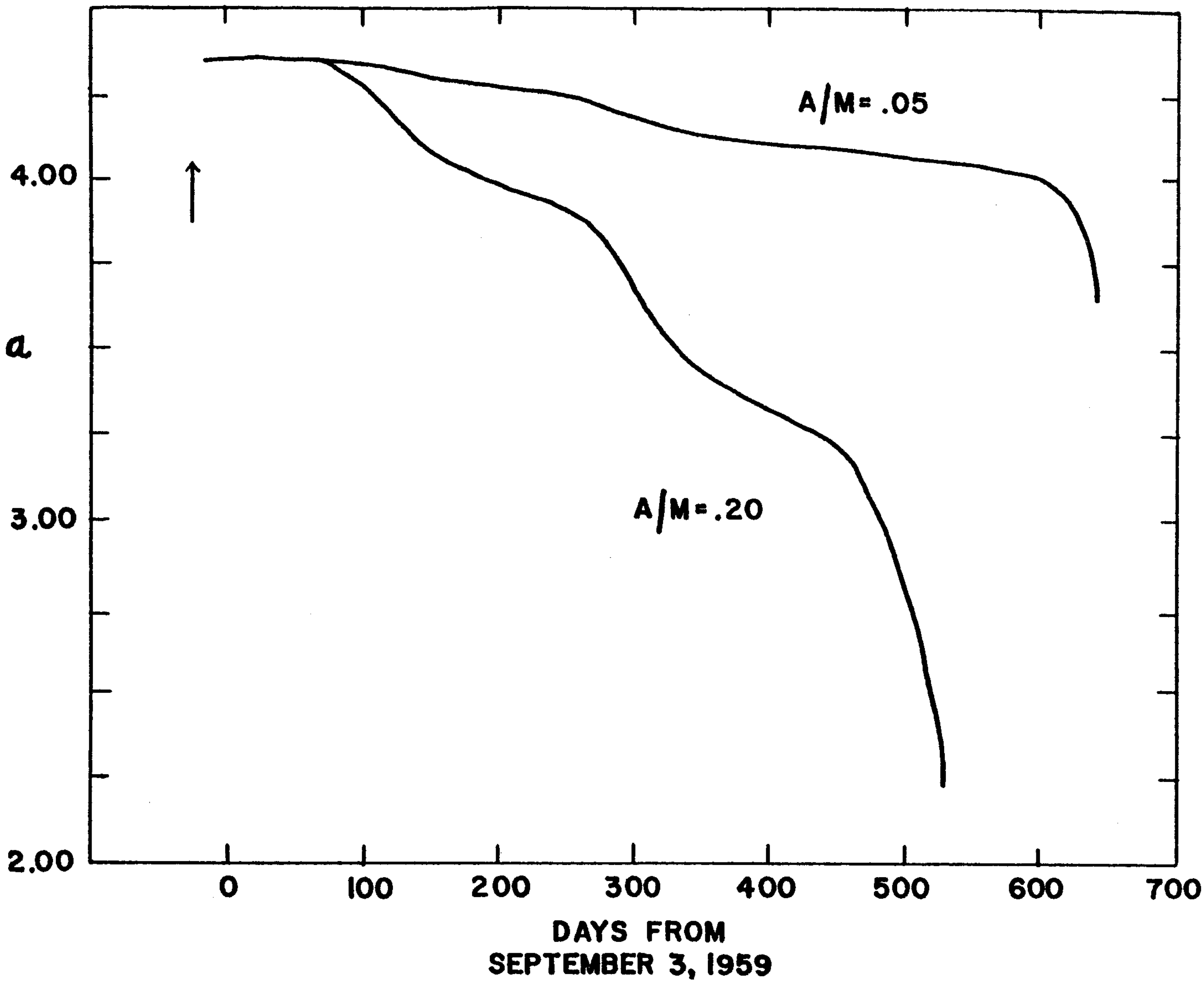

FIGURE 3. - EXPECTED SEMI-MAJOR AXIS OF 1959 DELTA TWO MEASURED IN EARTH RADII.

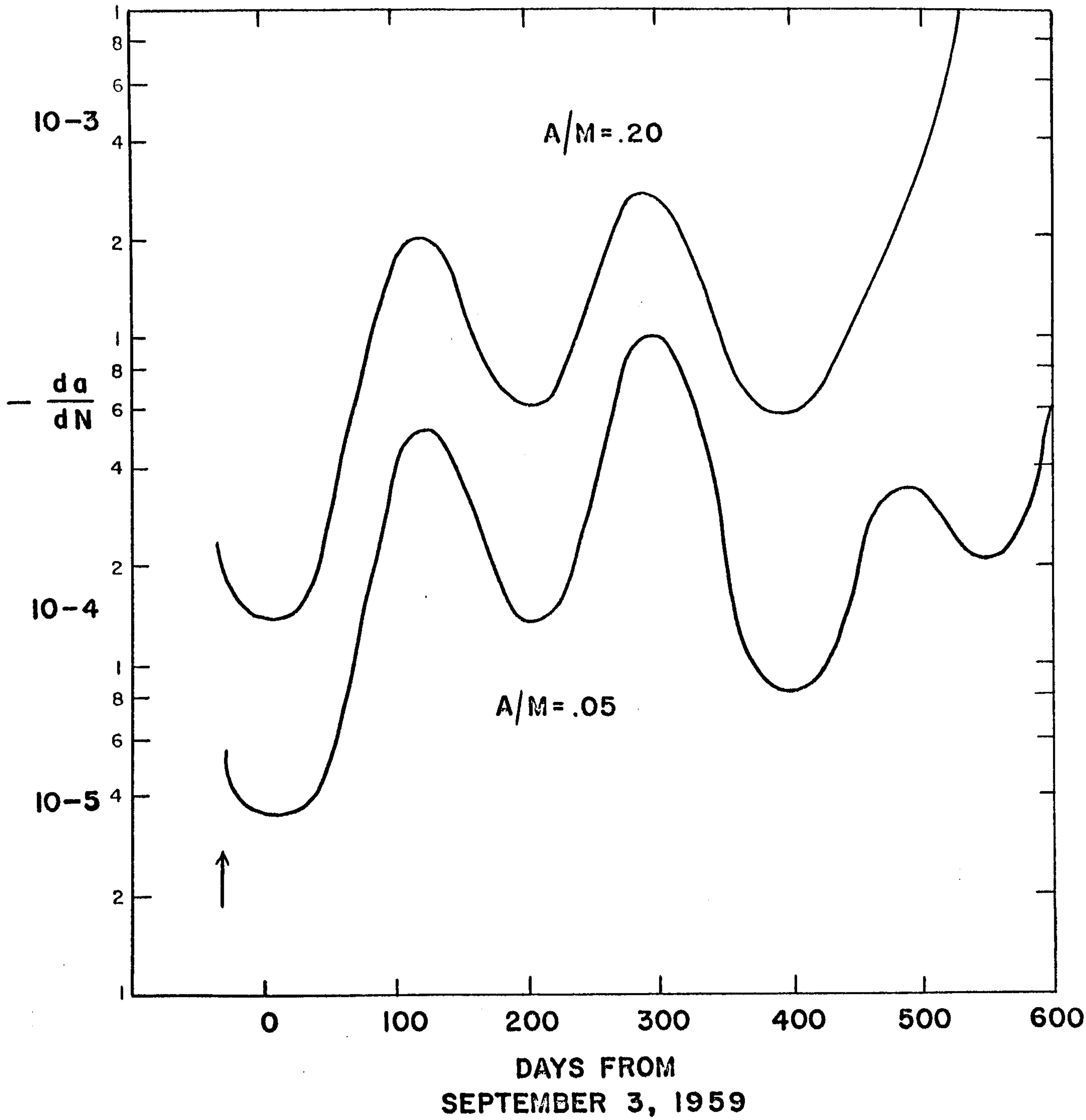

FIGURE 2. - EXPECTED ACCELERATION OF SATELLITE 1959 DELTA TWO GIVEN IN TERMS OF THE DECREASE OF SEMI-MAJOR AXIS (IN EARTH RADII) PER REVOLUTION.

Group 6 (N = 4500 to 5300)

$$P = .09310678 - 1.64166 \times 10^{-8} (N-4500) + 1.81251 \times 10^{-11} (N-4500)^2 - 1.6668 \times 10^{-14} (N-4500)^3$$

Group 7 (N = 5250 to 5600)

$$P = .09309703 - 1.100 \times 10^{-8} (N-5300) + 2.001 \times 10^{-11} (N-5300)^2$$

Expressions for the remaining parameters were derived from weekly orbit data supplied again by the NASA computing center. For all groups the following representations were used, where $T = t - 1958$ Oct. 27.0:

$$\omega = 32^\circ\!.200 + 4^\circ\!.40566T + 2^\circ\!.09574 \times 10^{-5} T^2 - 3^\circ\!.9506 \times 10^{-9} T^3 + {}^\circ\!.100 \cos \omega$$

$$i = 34^\circ\!.249 - {}^\circ\!.008 \sin \omega + {}^\circ\!.003 \sin \left(\frac{T-25}{47.75} \right)_{rad}$$

$$q = 1.10235 - 1.9 \times 10^{-6} T - .00060 \sin \omega + .00025 \sin \left(\frac{T+100}{57.30} \right)_{rad} \quad \text{(earth radii)}$$

The ascending node was represented by two separate expressions. For groups 1, 2, and 3,

$$\Omega = 200^\circ\!.573 - 3^\circ\!.01525T - 1^\circ\!.251 \times 10^{-5} T^2 + 2.667 \times 10^{-9} T^3 + {}^\circ\!.028 \cos \omega$$

and for groups 4, 5, 6, and 7,

$$\Omega = 200^\circ\!.569 - 3^\circ\!.013883T - 3^\circ\!.166 \times 10^{-5} T^2 + 3^\circ\!.483 \times 10^{-8} T^3 + {}^\circ\!.028 \cos \omega$$

Before May 6, 1958 there were sometimes gaps of several days with few or no observations. Furthermore, the orbits before this time were computed less frequently and are of generally poorer quality. For these reasons the earlier values of the passage times in Table 1 are given only to 5 decimal places.

References

JACCHIA, L. G., AND BRIGGS, R. E.

1958. Orbital acceleration of Satellite 1958 Beta Two. Special Rep. No. 18, Smithsonian Astrophys. Obs., pp. 9-12.

JACCHIA, L. G.

1958. Program for determination of geographic sub-satellite points. Special Rep. No. 11, Smithsonian Astrophys. Obs., p. 18.

A TABLE OF THE TIMES OF PERIGEE PASSAGE FOR SATELLITE 1958 BETA TWO

by

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The present table, which supersedes an earlier table (Jacchia and Briggs, 1958), is based on an analysis of nearly 4000 observations of Satellite 1958 β 2 made between March 17, 1958 and August 23, 1959. Nearly 95 percent of these are Minitrack observations,† almost 5 percent were obtained from Baker-Nunn photographs, while the remainder consist of Moonwatch observations. Each observation was used to compute a time of perigee passage, according to the sub-satellite point program of Jacchia (1958). Before performing this computation, however, this writer rewrote the entire program in a greatly expanded form for the IBM-704 Electronic Data Processing Machine. The new machine program permits the orbital parameters used for input to be represented by special functions such as those given below. After the individual times of perigee passage were computed, a graphical study of the results yielded well-determined curves from which the values in Table 1 were taken at regular intervals. The tabular values have been smoothed in 30 cases by $\pm$ .000001; no other smoothing was done. Since the second derivative, dP/dt, of these times of passage is of special interest to many investigators it is included in Table 1, and its behavior can be clearly seen in Figure 1.

In order to represent with sufficient accuracy the orbital parameters required by the computation program, the observations were divided into 7 overlapping groups according to the revolution number, N. Within each group a preliminary analysis using the machine program mentioned above showed that the anomalistic period, P, could be satisfactorily represented by the following expressions:

<u>Group 1</u> (N = 0 to 1000)

$$P = .09325678 - 1.21974 \times 10^{-8} N - 1.75314 \times 10^{-11} N^2 + 1.052 \times 10^{-14} N^3$$

<u>Group 2</u> (N = 900 to 1900)

$$P = .09323922 - 1.73808 \times 10^{-8} (N-909) + 1.83438 \times 10^{-11} (N-900)^2 - 2.2812 \times 10^{-14} (N-900)^3$$

<u>Group 3</u> (N = 1800 to 2800)

$$P = .09321965 - 1.58580 \times 10^{-8} (N-1800) - 7.6437 \times 10^{-11} (N-1800)^2 + 4.688 \times 10^{-14} (N-1800)^3$$

<u>Group 4</u> (N = 2700 to 3700)

$$P = .09317719 - 4.5234 \times 10^{-8} (N-2700) - 4.9062 \times 10^{-12} (N-2700)^2 + 7.396 \times 10^{-15} (N-2700)^3$$

<u>Group 5</u> (N = 3600 to 4600)

$$P = .09313776 - 2.9854 \times 10^{-8} (N-3600) - 1.96875 \times 10^{-11} (N-3600)^2 + 1.7708 \times 10^{-14} (N-3600)^3$$

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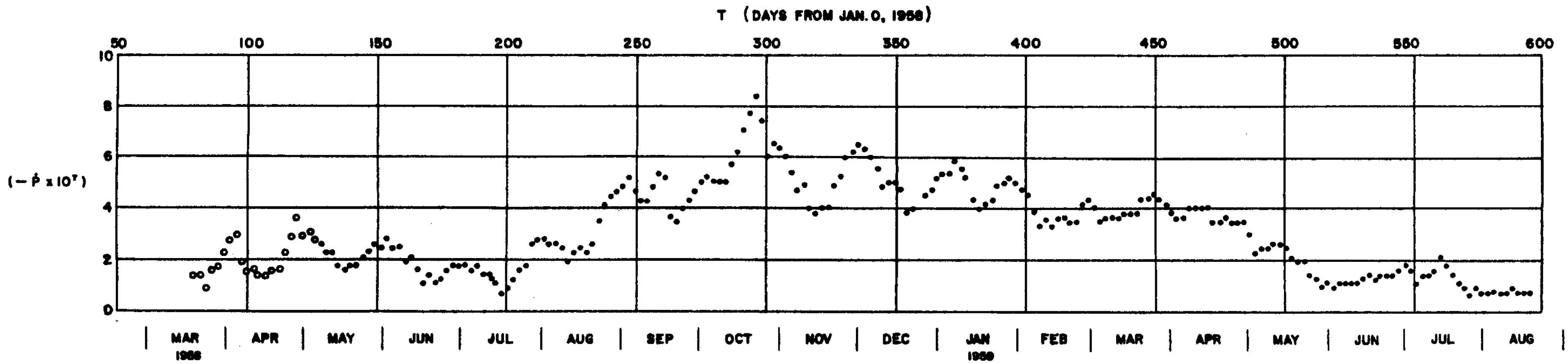
† Supplied by the NASA computing center.

Table 1 (continued)

N		T_π (UT)		$-P \times 10^7$
2850		8	.299768	5.5
2875		10	.628998	4.8
2900		12	.958200	5.0
2925		15	.287373	5.0
2950		17	.616517	4.6
2975		19	.945634	3.8
3000		22	.274729	4.0
3025		24	.603801	4.0
3050		26	.932850	4.5
3075		29	.261873	4.6
3100		31	.590869	5.2
3125	59 Jan.	2	.919835	5.3
3150		5	.248770	5.3
3175		7	.577674	5.8
3200		9	.906544	5.5
3225		12	.235382	5.2
3250		14	.564190	4.3
3275		16	.892973	4.0
3300		19	.221733	4.1
3325		21	.550469	4.3
3350		23	.879180	4.8
3375		26	.207863	5.0
3400		28	.536517	5.2
3425		30	.865141	5.0
3450	Feb.	2	.193736	4.6
3475		4	.522304	4.5
3500		6	.850846	3.8
3525		9	.179366	3.3
3550		11	.507867	3.4
3575		13	.836348	3.3
3600		16	.164810	3.6
3625		18	.493251	3.6
3650		20	.821671	3.4
3675		23	.150071	3.4
3700		25	.478451	4.1
3725		27	.806807	4.3
3750	Mar.	2	.135138	4.0
3775		4	.463446	3.4
3800		6	.791732	3.6
3825		9	.120001	3.6
3850		11	.448247	3.6
3875		13	.776472	3.8
3900		16	.104675	3.8
3925		18	.432856	3.8
3950		20	.761015	4.3
3975		23	.089149	4.3
4000		25	.417258	4.5
4025		27	.745341	4.3
4050		30	.073399	4.1
4075	April	1	.401433	3.8
4100		3	.729445	3.6
4125		6	.057436	3.6
4150		8	.385406	4.0
4175		10	.713353	4.0
4200		13	.041277	4.0
4225		15	.369178	4.0
4250		17	.697056	3.4
4275		20	.024914	3.4
4300		22	.352752	3.6
4325		24	.680569	3.4
4350		27	.008366	3.4
4375		29	.336143	3.4
4400	May	1	.663900	2.9
4425		3	.991640	2.2
4450		6	.319367	2.4
4475		8	.647080	2.4
4500		10	.974779	2.6
4525		13	.302463	2.6
4550		15	.630132	2.4
4575		17	.957787	2.1
4600		20	.285430	1.9
4625		22	.613062	1.9
4650		24	.940683	1.4
4675		27	.268296	1.2
4700		29	.595902	0.9
4725		31	.923503	1.0
4750	June	3	.251098	0.9
4775		5	.578688	1.0
4800		7	.906272	1.0
4825		10	.233850	1.0
4850		12	.561422	1.0
4875		14	.888988	1.2
4900		17	.216547	1.4
4925		19	.544098	1.2
4950		21	.871642	1.4
4975		24	.199178	1.4
5000		26	.526706	1.4
5025		28	.854226	1.6
5050	July	1	.181737	1.7
5075		3	.509238	1.6
5100		5	.836730	1.0
5125		8	.164216	1.4
5150		10	.491694	1.4
5175		12	.819164	1.6
5200		15	.146625	2.1
5225		17	.474074	1.7
5250		19	.801513	1.4
5275		22	.128944	1.0
5300		24	.456369	0.9
5325		26	.783789	0.5
5350		29	.111206	0.9
5375		31	.438618	0.7
5400	Aug.	2	.766026	0.7
5425		5	.093430	0.9
5450		7	.420830	0.7
5475		9	.748226	0.7
5500		12	.075618	0.9
5525		14	.403005	0.7
5550		16	.730388	0.7
5575		19	.057767	0.7
5600		21	.385142	

Table 1

N	T_π (UT)	$-P \times 10^7$ (days/day)	N	T_π (UT)	$-P \times 10^7$
0	58 Mar. 17 .61062		1425	28 .482903	2.6
25	19 .94203	1.4	1450	30 .813698	2.8
50	22 .27344	1.4	1475	Aug. 2 .144477	2.8
75	24 .60483	0.9	1500	4 .475240	2.6
100	26 .93623	1.5	1525	6 .805988	2.6
125	29 .26761	1.7	1550	9 .136721	2.4
150	31 .59898	2.2	1575	11 .467440	1.9
175	Apr. 2 .93034	2.8	1600	13 .798148	2.2
200	5 .26169	2.9	1625	16 .128843	2.4
225	7 .59301	1.9	1650	18 .459524	2.2
250	9 .92433	1.5	1675	20 .790192	2.6
275	12 .25564	1.5	1700	23 .120845	3.4
300	14 .58693	1.4	1725	25 .451478	4.1
325	16 .91823	1.4	1750	27 .782087	4.5
350	19 .24951	1.5	1775	30 .112670	4.6
375	21 .58078	1.5	1800	Sept. 1 .443226	4.8
400	23 .91204	2.2	1825	3 .773754	5.2
425	26 .24329	2.9	1850	6 .104252	4.6
450	28 .57453	3.6	1875	8 .434723	4.3
475	30 .90574	2.9	1900	10 .765169	4.3
500	May 3 .23693	3.1	1925	13 .095590	4.8
525	5 .56811	2.8	1950	15 .425983	5.3
550	7 .899270	2.6	1975	17 .756345	5.2
575	10 .230416	2.2	2000	20 .086677	3.6
600	12 .561549	2.2	2025	22 .416988	3.4
625	14 .892669	1.7	2050	24 .747279	4.0
650	17 .223779	1.5	2075	27 .077547	4.3
675	19 .554880	1.7	2100	29 .407790	4.6
700	21 .885971	1.7	2125	Oct. 1 .738006	5.0
725	24 .217052	2.1	2150	4 .068193	5.2
750	26 .548121	2.2	2175	6 .398351	5.0
775	28 .879177	2.6	2200	8 .728479	5.0
800	31 .210218	2.4	2225	11 .058578	5.0
825	June 2 .541245	2.7	2250	13 .388648	5.7
850	4 .872256	2.4	2275	15 .718685	6.2
875	7 .203253	2.5	2300	18 .048686	7.0
900	9 .534237	1.9	2325	20 .378646	7.7
925	11 .865210	2.1	2350	22 .708561	8.4
950	14 .196171	1.5	2375	25 .038427	7.4
975	16 .527123	1.0	2400	27 .368250	6.0
1000	18 .858069	1.4	2425	29 .698038	6.5
1025	21 .189007	1.0	2450	Nov. 1 .027788	6.4
1050	23 .519939	1.2	2475	3 .357501	6.0
1075	25 .850864	1.5	2500	5 .687179	5.3
1100	28 .181780	1.7	2525	8 .016826	4.6
1125	30 .512686	1.7	2550	10 .346446	4.8
1150	July 2 .843582	1.7	2575	12 .676038	4.0
1175	5 .174468	1.5	2600	15 .005607	3.8
1200	7 .505345	1.7	2625	17 .335154	4.0
1225	9 .836212	1.4	2650	19 .664678	4.0
1250	12 .167071	1.4	2675	21 .994179	4.8
1275	14 .497922	1.1	2700	24 .323652	5.2
1300	16 .828767	0.7	2725	26 .653095	6.0
1325	19 .159608	0.9	2750	28 .982503	6.2
1350	21 .490444	1.2	2775	Dec. 1 .311875	6.5
1375	23 .821273	1.5	2800	3 .641209	6.4
1400	26 .152093	1.7	2825	5 .970506	6.0


T (DAYS FROM JAN. 0, 1958)
50
100
150
200
250
300
350
400
450
500
550
600
10
8
6
4
2
0
$(-\dot{P} \times 10^7)$
MAR
1958
APR
MAY
JUN
JUL
AUG
SEP
OCT
NOV
DEC
JAN
1959
FEB
MAR
APR
MAY
JUN
JUL
AUG

NOTE ON THE SECULAR MOTIONS OF THE NODE AND PERIGEE OF AN ARTIFICIAL SATELLITE

by

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The values of the coefficients of the second and the fourth harmonics of the earth's gravitational potential, which I derived from the motion of the node and perigee of the first Vanguard Satellite 1958 β 2 (Kozai, 1959), are quite different from those obtained by other authors. Recently I discovered an error in my formula for the secular motion of the perigee as it appeared in the previous paper. Instead of

$$\omega = \frac{A_2}{p^2} n \,(4 - 5 \sin^2 i) \left\{ \frac{1}{2} + \frac{A_2}{a^2} (1 - \frac{31}{48} \sin^2 i) \right\}$$

we must read

$$\dot{\omega} = \frac{A_2}{p^2} n \,(4 - 5 \sin^2 i) \left\{ \frac{1}{2} + \frac{29}{48} \frac{A_2}{a^2} \sin^2 i \right\} .$$

This change should give more reasonable values for the coefficients. In the meantime, however, I have derived more complete formulae for the secular motions of the node and perigee, which are applicable to any satellite even if its eccentricity is not small. They are of closed form as far as the terms of order of A_2^2/a^4.

The new formulae are:

$$\dot{\Omega} = - \frac{A_2}{p^2} n \cos i \left[1 + \frac{A_2}{p^2} \left\{ \frac{3}{2} + \frac{e^2}{6} - 2\sqrt{1-e^2} - \sin^2 i \left(\frac{5}{3} - \frac{5}{24} e^2 - 3\sqrt{1-e^2} \right) \right\} \right]$$

$$- \frac{A_4}{p^4} n \cos i \left(\frac{6}{7} - \frac{3}{2} \sin^2 i \right) \left(1 + \frac{3}{2} e^2 \right) ,$$

$$\dot{\omega} = \frac{A_2}{p^2} n \left(2 - \frac{5}{2} \sin^2 i \right) \left[1 + \frac{A_2}{p^2} \left\{ 2 + \frac{e^2}{2} - 2\sqrt{1-e^2} - \sin^2 i \left(\frac{43}{24} - \frac{e^2}{48} - 3\sqrt{1-e^2} \right) \right\} \right]$$

$$- \frac{5}{12} \frac{A_2^2}{p^4} n \, e^2 \cos^4 i$$

$$+ \frac{A_4}{p^4} n \left[\frac{12}{7} - \frac{93}{14} \sin^2 i + \frac{21}{4} \sin^4 i + e^2 \left(\frac{27}{14} - \frac{189}{28} \sin^2 i + \frac{81}{16} \sin^4 i \right) \right]$$

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Here n is the anomalistic mean motion and $p = a(1-e^2)$. Here i and e are their mean values over an entire orbital period, and a is determined from the equation,

$$n^2 a^3 = k^2 M \left\{ 1 - \frac{A_2}{p^2} \sqrt{1-e^2} \left(1 - \frac{3}{2} \sin^2 i\right) \right\} .$$

Using the following data of Satellite 1958 β2 for Oct. 16, $12^h 27^m$(UT), 1958, as used in Special Report No. 22,

Ω = -3.014 66 degrees per day,

ω = 4.404 05 degrees per day,

n = 3862.640 degrees per day,

e = 0.190 00,

i = 34.250 degrees,

I derived as the values of A_2/a_e^2 and A_4/a_e^4:

$$\begin{cases} \dfrac{A_2}{a_e^2} = 1.6232 \times 10^{-3} , \\ \\ \dfrac{A_4}{a_e^4} = 0.94 \times 10^{-5} . \end{cases}$$

These values are reasonable and are not very different from those by other authors.

References

KOZAI, Y.
1959. The earth's gravitational potential derived from the motion of Satellite 1958 Beta Two. Special Rep. No. 22, Smithsonian Astrophys. Obs., pp. 1-6.

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